AP Calculus BC

Summer Packet

Due August 28, 2018

**Please complete neatly and show all work!*

TO: AP Calculus BC Students
FROM: Mr.Blitshteyn , AP Calculus BC Teacher

Attached is a summer homework packet, which will need to be completed by first day of Calculus class in September. The material in the packet should be material you learned in Algebra II, Precalculus, and Calculus AB.

My recommendation is that you look over the problems in the packet when you receive it and start to work on them. However, then you wait until the two-three weeks before school starts to work the problems, so in this case you will remember the material very well when school starts.

You will be expected to have a TI-83, TI-84 calculators.
I am looking forward to seeing you in Calculus in September.

Sincerely,

Mr. Blitshteyn
Bishop Connolly High School, Math Department

CALCULUS BC
SUMMER NOTES

Some of the material that was briefly covered in Pre-Calculus is knowledge that I need you to know to be able to complete the packet. Here are some notes/examples for you to use:

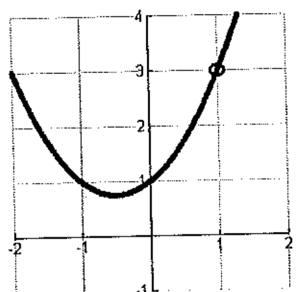
The Limit:

1) Let's take a look at the function $f(x) = \frac{x^3 - 1}{x - 1}$.

What do we know about the graph of this function? There is a hole at x=1

Right!

Graph it.



➔If we put the point of our pencil on f (0.1) and trace the curve towards f (1) our pencils approaches the y-value of 3.

We denote this as: $\lim_{x \to 1^-} f(x) = 3$ because as we approach f (1) from the left side (the reason for the raise – next to the 1) the value of the graph approaches 3.

➔If we put the point of our pencil on f (1.3) and trace the curve towards f (1) our pencils approaches the y-value of 3.

We denote this as: $\lim_{x \to 1^+} f(x) = 3$ because as we approach f (1) from the right side (the reason for the raised + next to the 1) the value of the graph approaches 3.

➔Because the limit from the left side **is equal to** the limit from the right side, we can use the notation: $\lim_{x \to 1} f(x) = 3$ (notice the lack of a raised + or –). If this is not the case, we say the limit does not exist.

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2) We can also evaluate simple limits quite easily, without even looking at the graph.

Ex 1: $\lim_{x \to 3} x^2 + 3x - 1$

➔ If we first think about the function $f(x) = x^2 + 3x - 1$ we know that it's graph does not have any special properties (holes / asymptotes/ etc.)

➔ <u>substitute</u> the x-value we are approaching with the x-values in the function.

$\lim_{x \to 3}(x^2 + 3x - 1) = 3^2 + 3(3) - 1 = 17$

Therefore, $\lim_{x \to 3}(x^2 + 3x - 1) = 17$.

Ex 2: $\lim\limits_{x\to 2}\dfrac{x^2+2x-8}{x-2}$

➔ If we first think about the function $f(x)=\dfrac{x^2+2x-8}{x-2}$ we know there exists a hole at x = 2 and that happens to be the x-value we want to know about, so our previous method won't work. So we have to be a little more creative

➔ Think about the <u>factors</u> of x^2+2x-8. One of the factors is (x – 2).

$\lim\limits_{x\to 2}\dfrac{x^2+2x-8}{x-2}=\lim\limits_{x\to 2}\dfrac{(x-2)(x+4)}{x-2}=\lim\limits_{x\to 2}x+4$ Now we can substitute like we did on problem #1.

$\lim\limits_{x\to 2}(x+4)=6$

Therefore, $\lim\limits_{x\to 2}\dfrac{x^2+2x-8}{x-2}=6$

Ex 3: $\lim\limits_{x\to 0}\dfrac{\sqrt{x+2}+1}{x}$

➔ Try <u>using the conjugates</u>

$$\lim_{x\to 0}\frac{\sqrt{x+9}+9}{x}\square\frac{\sqrt{x+9}-9}{\sqrt{x+9}-9}=\lim_{x\to 0}\frac{x+9-9}{x\left(\sqrt{x+9}-9\right)}=\lim_{x\to 0}\frac{x}{x\left(\sqrt{x+9}-9\right)}=\lim_{x\to 0}\frac{1}{\left(\sqrt{x+9}-9\right)}=-\frac{1}{6}$$

Ex 4: $\lim\limits_{x\to\infty}\dfrac{x^3+2x^2-5}{3x-4x^3+2}$

➔ If the limit is going off to positive or negative infinity, it is asking you to find the <u>horizontal asymptote</u>. So remember your methods to finding the horizontal asymptote. (Ratio of the leading coefficients)

$$\lim_{x\to\infty}\frac{x^3+2x^2-5}{3x-4x^3+2}=\lim_{x\to\infty}\frac{x^3+2x^2-5}{-4x^3+3x+2}=-\frac{1}{4}$$

Note: To complete the optimization problems, you should have learned how to use a system of equations, the substitution method, and your calculator to find the maximum or minimum value.

Ex 5: Find two positive numbers whose product is 115 and whose sum is a minimum.
Equation 1: $xy=115$ Equation 2: $S=x+y$, where x represents my first number, y represents my second number, and S represents my sum function.

Combine equations by solving for x or y in equation 1, you get: $S=x+\dfrac{115}{x}$

Put S into Y1 and use calculator to find the minimum x-value. For the window, you can put (0, 50) for X, and (-50, 50) for Y. Then use that x-value in equation 1 to solve for the y-value. In this case,
X = 10.724 and Y = 21.448

CALCULUS BC
SUMMER HOMEWORK

This homework packet is due the first day of school. It will be turned in the first day of Calculus class and will count as a daily grade.

Work these problems on <u>notebook paper</u>. All work must be shown.

Find the x- and y-intercepts and the domain and range, and sketch the graph.
Do <u>not</u> use your graphing calculator on these.

1. $y=2-x^2$
2. $y=(x+2)^2$
3. $y=\sqrt{x-1}$
4. $y=\sqrt{9-x^2}$
5. $y=x^3+2$
6. $y=\frac{|x|}{x}$
7. $y=\sin x,\ -2\pi\le x\le 2\pi$
8. $y=\cos x,\ -2\pi\le x\le 2\pi$
9. $y=\tan x,\ -2\pi\le x\le 2\pi$
10. $y=e^x$
11. $y=\ln x$
12. $y=\begin{cases}-1 & ,\ if\ x\le -1\\ 3x+2, & if\ |x|<1\\ 7-2x, & if\ x\ge 1\end{cases}$
13. $y=\begin{cases}x^2+1, & if\ x>0\\ -2x+2, & if\ x\le 0\end{cases}$

Find the asymptotes (horizontal, vertical, and slant), symmetry, and intercepts, and sketch the graph. Do <u>not</u> use your graphing calculator on these.

14. $y=\frac{1}{x-1}$
15. $y=\frac{1}{(x+2)^2}$
16. $y=\frac{2(x^2-9)}{x^2-4}$
17. $y=\frac{x^2-2x+4}{x-1}$

Solve.

18. $x^2-x-12>0$
19. $(x-2)^2(x+1)^3(x-5)\le 0$
20. $\frac{3x-2}{x+4}\le 0$
21. $\frac{(2x+5)(x-1)^2}{(x+2)^3}\ge 0$

Find the limit.

22. $\lim_{x\to 2}\left(4x^2-5x+3\right)$

23. $\lim_{x\to 0}\frac{\sqrt{x+1}-1}{x}$

24. $\lim_{x\to 0}\sqrt{x^2+4}$

25. $\lim_{x\to 0}\frac{(x-5)^2-25}{x}$

26. $\lim_{x\to -3}\frac{x^2+x-6}{x+3}$

27. $\lim_{x\to \infty}\frac{3x-5x^2}{4x^2+1}$

28. $\lim_{x\to -6}\frac{x+6}{x^2+3x-18}$

29. $\lim_{x\to -\infty}\frac{2+3x^3}{x^2+4}$

30. $\lim_{x\to -2}\frac{x^3+8}{x+2}$

Use the figure to find the limit.

31. $\lim_{x\to 3} f(x)$

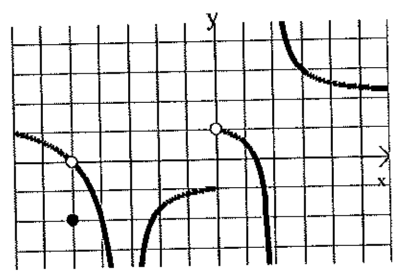
32. $\lim_{x\to 0} f(x)$

33. $\lim_{x\to \infty} f(x)$

34. $\lim_{x\to -\infty} f(x)$

35. $\lim_{x\to 2^+} f(x)$

36. $\lim_{x\to -5} f(x)$



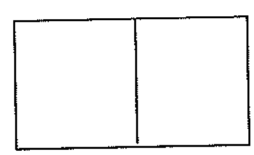
Graph of f

Write a function for each problem, and use your graphing calculator to solve. Give decimal answers correct to three decimal places. Be sure to sketch the graph you used, and label it with your answer.

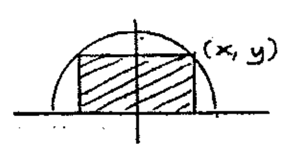
37. Find two positive numbers such that their product is 192 and their sum is a minimum.

38. Find two positive numbers such that their product is 192 and the sum of the first plus three times the second is a minimum.

39. A rancher has 200 feet of fencing with which to enclose two adjacent rectangular corrals, as shown. What dimensions should be used so that the enclosed area will be a maximum?



40. A rectangle is bounded by the x-axis and the semicircle $y=\sqrt{25-x^2}$ as shown. What length and width should the rectangle have so that its area is a maximum?



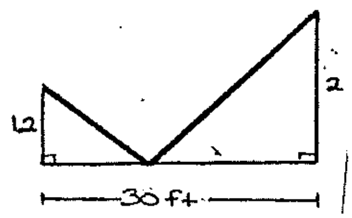
41. A rectangular pace is to contain 24 square inches of print. The margins at the top and bottom of the page are to be $1\frac{1}{2}$ inches, and the margins on the left and right are to be 1 inch. What should the dimensions of the page be so that the least amount of paper

is used?

42. Four feet of wire is to be used to form a square and a circle. How much of the wire should be used for the square and how much should be used for the circle to enclose the maximum total area?

43. The sum of the perimeters of an equilateral triangle and a square is 10. Find the dimensions of the triangle and the square that produce a minimum total area.

44. Two posts, one 12 feet high and the other 28 feet high, stand 30 feet apart. They are to be stayed by two wires, attached to a single stake, running from ground level to the top of each post. How far from the left post should the stake be placed to use the least wire?



Fill out the table:

Angle(radians)	Cos	Sin	Sec	Csc	tan	Cot
0						
$\frac{\pi}{6}$						
$\frac{\pi}{4}$						
$\frac{\pi}{3}$						
$\frac{\pi}{2}$						

Solve. Give exact answers in radians, $0 \le x \le 2\pi$.

45. $2\cos x + \sqrt{3} = 0$

46. $2\cos^2 x + 3\cos x - 2 = 0$

47. $\sin 2x = -1$

48. $2\cos^2 x + \sin x - 1 = 0$

49. $\sin 2x = \cos x$

50. $\tan^2 x - \sec x = 1$

Know the following Trigonometric Identities:

Pythagorean: $\sin^2 x + \cos^2 x = 1$ $\quad 1 + \tan^2 x = \sec^2 x$ $\quad 1 + \cot^2 x = \csc^2 x$

Division: $\sec x = \dfrac{1}{\cos x}$ $\quad \csc x = \dfrac{1}{\sin x}$ $\quad \cot x = \dfrac{1}{\tan x}$ $\quad \tan x = \dfrac{\sin x}{\cos x}$ $\quad \cot x = \dfrac{\cos x}{\sin x}$

Double Angle: $\sin(2x) = 2\sin x \cos x$

Product to Sum: $\sin x \cdot \sin y = \frac{1}{2}[\cos(x-y) - \cos(x+y)$

$$\cos x \cdot \cos y = \frac{1}{2}[\cos(x-y)+\cos(x+y)$$
$$\sin x \cdot \cos y = \frac{1}{2}[\sin(x+y)-\sin(x-y)$$

Logarithms:

Inverse Property: $\log_b y = x$ *iff* $b^x = y$

Recall: $\ln x = \log_e x$

Product Property: $\log_b(mn) = \log_b m + \log_b n$

Quotient Property: $\log_b(\frac{m}{n}) = \log_b m - \log_b n$

Power Property: $\log_b(m^n) = n \cdot \log_b m$

Expand the following:

51. $\log_4 \sqrt{(3m)}$

52. $\log_9(27m^2)$

53. $\ln\left(\frac{x+4}{3x}\right)$

Solve the following log and exponential equations for x:

54. $3^x = 3^{x-4}$

55. $4^x = 16$

56. $3^x = 9^{x+1}$

57. $\left(\frac{1}{2}\right)^x = 4$

58. $\log_4 x = 3$

59. $\log_2 8 = x$

60. $\log_3 9 = x+1$

61. $2\log_3 x = 8$

62. $\log_4(x+3) = 2$