Precalculus CP

Summer Packet

Due August 28, 2018

**Please complete neatly and show all work!*

Math Help Websites:

The internet has a vast amount of resources. To utilize your search in an efficient manner, enter the specific topic skill listed below to narrow your results.

http://www.coolmath.com
http://www.aplusmath.com
http://www.mathplayground.com
http://www.purplemath.com
http://www.ixl.com
http://softschools.com
http://math.com/homeworkhelp
http://kidsmathgamesonline.com
http://learnzillion.com
www.mrmathblog.com

http://www.khanacademy.org (great videos to help you if you get stuck)!

Perform the indicated operation. Show ALL work.

$3\frac{1}{2} + 6\frac{2}{3}$

$2\frac{3}{5} + 10\frac{1}{7}$

$9\frac{2}{9} + 5\frac{3}{8}$

$5\frac{2}{3} - 2\frac{1}{4}$

$6\frac{4}{5} - 2\frac{9}{10}$

$8 - 5\frac{4}{7}$

$4 \bullet \frac{6}{11}$

$\frac{2}{3} \bullet \frac{5}{9}$

Perform the indicated operation. Show ALL work.

$3\frac{1}{5} \bullet \frac{3}{8}$

$\frac{4}{9} \bullet 2\frac{1}{7}$

$8\frac{1}{3} \div 5$

$\frac{9}{14} \div \frac{2}{3}$

$6\frac{1}{4} \div 2\frac{1}{8}$

RATIONALIZING DENOMINATORS

Objective: Simplify a radical with an irrational denominator.

Example 1

Simplify

$$\frac{2}{\sqrt{3}}$$

$$\frac{2}{\sqrt{3}} \bullet \frac{\sqrt{3}}{\sqrt{3}}$$

$$\frac{2 \bullet \sqrt{3}}{\sqrt{3} \bullet \sqrt{3}}$$

$$\frac{2\sqrt{3}}{\left(\sqrt{3}\right)^2}$$

$$\frac{2\sqrt{3}}{3}$$

Example 2

Simplify

$$\sqrt{\frac{5}{8}}$$

$$\frac{\sqrt{5}}{\sqrt{8}}$$

$$\frac{\sqrt{5}}{\sqrt{8}} \bullet \frac{\sqrt{8}}{\sqrt{8}}$$

Remember to rationalize the denominator, multiply the numerator and denominator by the radical you want to eliminate.

$$\frac{\sqrt{40}}{8}$$ Simplify the radical

$$\frac{2\sqrt{10}}{8}$$

$$\frac{\sqrt{10}}{4}$$

Rationalize each denominator. Show ALL work.

66. $\frac{3}{\sqrt{5}}$

67 $\frac{4}{\sqrt{6}}$

68. $\frac{1}{\sqrt{7}}$

69. $\sqrt{\frac{3}{8}}$

70. $\sqrt{\frac{2}{3}}$

71. $\sqrt{\frac{1}{9}}$

72. $\sqrt{\frac{24}{11}}$

73. $\frac{9}{\sqrt{5}}$

74. $\frac{10}{\sqrt{8}}$

75. $\frac{12}{5\sqrt{3}}$

76. $\frac{20}{6\sqrt{5}}$

LINES IN THE PLANE

You should know the following important facts about lines.

- The graph of $y = mx + b$ is a straight line. It is called a linear equation.
- The slope of the line through $(x_1, y_1) and (x_2, y_2)$ is $m = \frac{y_2 - y_1}{x_2 - x_1}$.
- If $m > 0$, the line rises from left to right.
- If $m < 0$, the line falls from left to right.
- If $m = 0$, the line is horizontal.
- If m is undefined, the line is vertical.
- Equations of Lines
  - Slope-Intercept: $y = mx + b$
  - Point-Slope: $y - y_1 = m(x - x_1)$
  - Two-Point: $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$
  - General: $Ax + By + C = 0$
  - Vertical: $x = a$
  - Horizontal: $y = b$
- Given two distinct non-vertical lines:
  - $L_1 = m_1 x + b_1$
  - $L_2: y = m_2 x + b_2$
  - L_1 is parallel to L_2 if and only if $m_1 = m_2$ and $b_1 \neq b_2$.
  - L_2 is perpendicular to L_2 if and only if $m_1 = \frac{-1}{m_2}$.

Find the slope of the line that passes through the points.

1. $(-3, 2), (8, 2)$

2. $(7, -1), (7, 12)$

3. $(\frac{3}{2}, 1), (5, \frac{5}{2})$

4. $(-\frac{3}{4}, \frac{5}{6}), (\frac{1}{2}, -\frac{5}{2})$

Use the concept of slope to find t such that the three points are collinear.

5. (-2 , 5) , (0 , t) , (1 , 1)

6. (-6 , 1) , (1 , t) , (10 , 5)

Find an equation of the line that passes through the given point and has the specified slope.

7. Point (2 , -1), Slope $m = \frac{1}{4}$.

8. Point (3 , 0), Slope $m = \frac{-2}{3}$.

9. Point (-2 , 6), Slope m = 0.

10. Point (5 , 4), Slope m is undefined.

Find an equation of the line(in slope-intercept form) that passes through the points.

11. (2 , -1) , (4 , -1)   12. (-1 , 0) , (6 , 2)   13. (1 , 6) , (4 , 2)

Write the equations of the lines through the point a) parallel, and b) perpendicular to the given line.

14. Point (3 , -2) , Line: $5x - 4y = 8$ 15. Point (-8 , 3), Line $2x + 3y = 5$

16. Point (-6 , 2), Line: $x = 4$ 17. Point (3 , -4), Line: $y = 2$

SOLVING INEQUALITIES ALGEBRAICALLY AND GRAPHICALLY

- You should know the properties of inequalities.
  - Transitive: $a < b$ and $b < c$ implies $a < c$.
  - Addition: $a < b$ and $c < d$ implies $a+c < b+d$
  - Adding or Subtracting a Constant: $a \pm c < b \pm c$ if $a < b$.
  - Multiplying or Dividing by a constant: For $a < b$,
    - If $c > 0$, then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$.
    - If $c < 0$, then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$.
- You should know that $|x| = \{ x$ if $x \geq 0$, $-x$ if $x < 0$.
- You should be able to solve absolute value inequalitites,
  - $|x| < a$ if and only if $-a < x < a$.
  - $|x| > a$ if and only if $x < -a$ or $x > a$.
- You should be able to solve polynomial inequalities.
  - Find the critical numbers.
    - Values that make the expression zero.
    - Values that make the expression undefined.
  - Test one value in each interval on the real number line resulting from the critical numbers.
  - Determine the solution intervals.
- You should be able to solve rational and other types of inequalities.

Solve the inequality and graph the solution on the real number line.

1. $8x - 3 < 6x + 15$

2. $-2 < -x + 7 \leq 10$

3. $|x-2| < 1$

4. $|x-3| > 4$

5. $4|3-2x| \leq 16$

6. $|x+9|+7>19$

7. $|x| \geq 4$

8. $\left|x-\frac{3}{2}\right| \leq \frac{3}{2}$

SOLVING PROPORTIONS

Objective: To solve a proportion using cross-multiplication.

Example

Solve for x.

a. $\frac{x}{4} = \frac{21}{7}$

$\frac{x}{4} \times \frac{21}{7}$ (Cross Multiply)

$x \cdot 7 = 4 \cdot 21$

$\frac{7x}{7} = \frac{84}{7}$ (Simplify)

$x = 12$

The solution set is $\{12\}$.

b. $\frac{3}{8} = \frac{-6}{4a}$

$\frac{3}{8} \times \frac{-6}{4a}$

$3 \cdot 4a = 8 \cdot (-6)$

$\frac{12a}{12} = \frac{-48}{12}$

$a = -4$

The solution set is $\{-4\}$.

Solve each proportion using cross multiplication. Leave answers in fraction form. Show ALL work.

1. $\frac{-8}{1} = \frac{4b}{5}$

2. $\frac{4r}{3} = -12$

3. $\frac{9}{1} = \frac{2n-1}{5}$

4. $\frac{6}{4x+3} = \frac{4}{2-5x}$

5. $\frac{x+6}{8} = \frac{x-6}{9}$

6. $\frac{x-6}{4} = \frac{x-9}{2}$

FACTORING POLYNOMIALS

Objective: a. To factor a polynomial using the greatest common factor (GCF).
b. To factor a difference of two squares.

Example for Objective a.

Factor.

$2x^3+4x^2$ (2x² is the greatest common factor.)

$2x^2(x+2)$ (Factor out the 2x² that all the terms have in common.)

In order for a polynomial to be a perfect square it must meet <u>three</u> conditions:

1. there are only two terms
2. each term is a perfect square
3. it <u>must</u> have a ***minus sign***

Example 1 for Objective b.

Factor

$x*x$ $5*5$

$x^2 - 25$ **Find the square root of each factor**

$(x+5)(x-5)$ **Follow the factoring pattern**

Example 2 for Objective b.

Factor

$2x*2x$ $9*9$

$4x^2 - 81$ **Find the square root of each factor**

$(2x+9)(2x-9)$ **Follow the factoring pattern**

Factor each polynomial completely. Show ALL work.

1. $3x^2-24x$
2. $4x^2+16x-32$
3. x^2-49
4. $16x^2-25$
5. $3x^3-12x$
6. $2x^2-128$
7. $25x^3-36x^2y$
8. $24x^2y^2+16x^3y-96xy$
9. $2x^4-162$

FACTORING POLYNOMIALS

Objective: To factor a trinomial.

Factoring Patterns for $x^2+bx+c=0$ when c is negative: $(x+\_\_)(x-\_\_)$

Example 1

Factor.

$x^2+12x-45$

1. Since the coefficient of the x^2 term is one, just think of factors of your last term: -45 that add up to the middle term: +12.
2. The factors of -45 that add up to +12 are +15 and -3 .
3. Therefore:

$x^2+12x-45=(x+15)(x-3)$

So, the solution is: $(x+15)(x-3)$

**Hint: To check your answer, simply use the foil method. If you come up with the trinomial that you started with, you are correct!

Factoring Patterns for $x^2+bx+c=0$ when c is positive:

When b is positive: $(x+\_\_)(x+\_\_)$

When b is negative: $(x-\_\_)(x-\_\_)$

Example 2

Factor.

$x^2-10x+16$

1. Since the coefficient of the x^2 term is one, just think of factors of your last term: +16 that add up to the middle term: -10.
2. The factors of +16 that add up to -10 are -8 and -2.
3. Therefore:

$x^2-10x+16=(x-2)(x-8)$

So, the solution is: $(x-2)(x-8)$

**Hint: To check your answer, simply use the foil method. If you come up with the trinomial that you started with, you are correct!

Factor Completely. Show ALL work.

1. x^2-2x-3
2. $x^2-11x+24$
3. $x^2+17x+30$
4. $x^2+5x-14$
5. $x^2+19x+60$
6. $x^2-10x+16$
7. $x^2-2x-35$
8. $2x^2+15x+7$
9. $3x^2+16x-44$

FACTORING TO SOLVE A QUADRATIC EQUATION

Objective: To solve a quadratic equation by factoring and using the Zero-Product Property.

Example

a. Use the Zero-Product Property; If $ab = 0$, then $a = 0$ or $b = 0$.

$x^2 - 7x + 10 = 0$	(Original equation)
$(x-5)(x-2) = 0$	(Factor)
$(x-5) = 0$ or $(x-2) = 0$	(Zero-Product Property)
$x = 5$ or $x = 2$	(Solve for x)

b. Use the Zero-Product Property; write the equation in standard form, $ax^2 + bx + c = 0$.

$2x^2 = 13x + 7$	(Original equation)
$2x^2 - 13x - 7 = 0$	(Write in standard form)
$(2x+1)(x-7) = 0$	(Factor)
$(2x+1) = 0$ or $(x-7) = 0$	(Zero-Product Property)
$x = -\frac{1}{2}$ or $x = 7$	(Solve for x)

Solve the equation by factoring and using the Zero-Product Property. Show ALL work.

1. $x^2 - 3x - 18 = 0$

2. $x^2 + 12x + 27 = 0$

3. $3x^2 - 13x - 10 = 0$

4. $2x^2 = 11x - 12$

5. $x^2 + 13x = -36$

6. $2x^3 - 6x^2 = 0$

Solving Quadratics Using the Quadratic Formula

OBJECTIVE: The student will be able to determine the number of real solutions in a quadratic equation using the discriminant and calculate the real solutions.

NOTES

Quadratic Formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

1) Keep in mind before applying the quadratic formula the equation must be in standard form, $ax^2+bx+c=0$, and when substituting the values in for a,b and c use only the coefficients not the variables. One part of the quadratic formula is called the which is the portion underneath the radical symbol.

Discriminant $= b^2 - 4ac$

The discriminant tells us how many real solutions there are in the original equation.. Use the following chart to determine the number of real solutions:

If the Discriminant >0 then there are 2 real solutions
If the Discriminant =0 then there is 1 real solution
If the Discriminant <0 then there are no real solutions

Example 1 **Use the discriminant to determine the number of real solutions of $-4x^2+x-14=0$**

$b^2 - 4ac$	Since the equation is in standard form, a=-4, b=1 and c=-14
$(1)^2-4(-4)(-14)$	Substitute the values into the discriminant
1-224	Simplify the 4ac portion
-223	Simplify the expression, note the discriminant is <0

Answer: no real solutions since the discriminant is less than zero

2) The quadratic formula is a powerful tool because it can solve any quadratic equation in standard form, $ax^2+bx+c=0$.

Example 3 Solve the equation $x^2+7x+11=0$

$(7)^2-4(1)(11)$ 5	Since the equation is in standard form, calculate the discriminant Discriminant equals 5, thus there are 2 real solutions we need to calculate.
$x = \frac{-7 \pm \sqrt{5}}{2(1)}$	Substitute the values into the quadratic formula

Answer $x = \frac{-7+\sqrt{5}}{2}$ *and* $\frac{-7-\sqrt{5}}{2}$

Directions: Determine the number of real solutions for the following equations.

1. $x^2-8x+16=0$
2. $8x^2+8x+3=0$
3. $5x^2+20x+21=0$
4. $8x-10=x^2-7x+3$

Directions: Solve the following equations using the quadratic equation

5. $x^2+8x+19=0$
6. $3x^2-12x=-12$
7. $8x^2-8x=2=0$
8. $5x^2-10x+24=0$

SIMPLIFYING RADICALS

Objective: To simplify radicals.

Example

a. Simplify $\sqrt{63}$

Identify factors that have perfect squares.

For example 9 has a perfect square, 3.

$\sqrt{63} = \sqrt{9 \cdot 7}$

$= \sqrt{9} \cdot \sqrt{7}$

$= 3 \cdot \sqrt{7}$

$= 3\sqrt{7}$

b. Simplify $2\sqrt{48x^4}$

$2\sqrt{48} = 2\sqrt{16 \cdot 3 \cdot x^4}$

$= 2\sqrt{16} \cdot \sqrt{3} \cdot \sqrt{x^4}$

$= 2 \cdot 4x^2\sqrt{3}$

$= 8x^2\sqrt{3}$

16 is a perfect square. Its square root is 4.

Divide the exponent by 2 since you're taking the square or 2nd root.

Simplify each radical completely. Show ALL work.

1. $\sqrt{108}$

2. $\sqrt{125}$

3. $\frac{1}{2}\sqrt{44x^2}$

4. $\sqrt{72}$

5. $5\sqrt{200}$

6. $\sqrt{242}$

7. $3\sqrt{144}$

8. $3\sqrt{50y^4}$

9. $20\sqrt{8x}$

Example 2

Simplify $\sqrt{\frac{9}{10}}$

$$\sqrt{\frac{9}{10}} = \frac{\sqrt{9}}{\sqrt{10}}$$

$$= \frac{\sqrt{9}\cdot\sqrt{10}}{\sqrt{10}\cdot\sqrt{10}}$$ (Rationalize the denominator to simplify)

$$= \frac{3\sqrt{10}}{10}$$

Simplify the radical completely. Leave no radicals in the denominator. Show ALL work.

10. $\frac{\sqrt{15}}{\sqrt{5}}$

11. $\sqrt{\frac{64}{7}}$

12. $\frac{\sqrt{30}}{\sqrt{6}}$

13. $\frac{\sqrt{18}}{\sqrt{10}}$

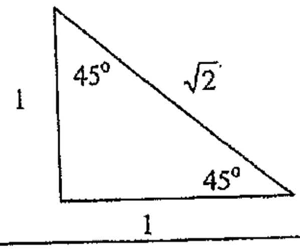
14. $\sqrt{\frac{81}{5}}$

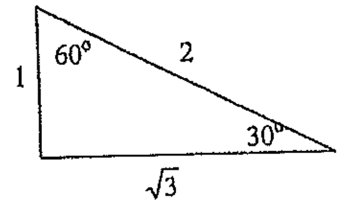
15. $\frac{\sqrt{26}}{\sqrt{2}}$

Applying Special Right Triangles

OBJECTIVE: The student will be able to calculate side lengths and angle measurements of special right triangles.

NOTES





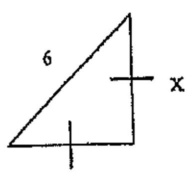
* When given a leg multiply the leg by $\sqrt{2}$ to calculate the hypotenuse.

* When given the hypotenuse divide by $\sqrt{2}$ to calculate the legs

* When given the short leg multiply the short leg by 2 to calculate the hypotenuse and multiply the short leg by $\sqrt{3}$ to calculate the long leg.

* If given the hypotenuse or long leg, calculate the short leg first by dividing by 2 when given the hypotenuse or dividing by $\sqrt{3}$ if given the long leg.

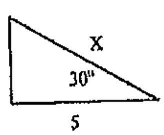
Example 1 Calculate the value of x for the triangle shown



We identify the triangle as a 45-45-90 because of the congruent hashes.
Since we are given the hypotenuse we will divide 6 by $\sqrt{2}$.

$$x = \frac{6}{\sqrt{2}} = \frac{6\sqrt{2}}{2} = 3\sqrt{2}$$

Example 2 Calculate the value of x for the triangle shown



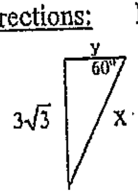
We identify the triangle as a 30-60-90.
Since we are given the long leg we will first divide by $\sqrt{3}$ to calculate the short leg

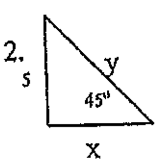
$$\text{Short leg} = \frac{5}{\sqrt{3}} = \frac{5\sqrt{3}}{3}$$

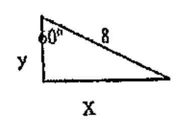
Now multiply the short leg by 2 to calculate the hypotenuse.

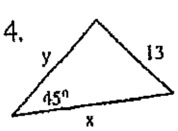
$$x = \frac{5\sqrt{3}}{3} \bullet 2 = \frac{10\sqrt{3}}{3}$$

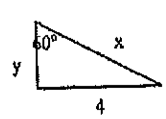
<u>Directions:</u> Find the exact values of x and y.

1. 

2. 

3. 

4. 

5. 

6.